QP CODE: 25025041

25025041

Reg No : .....

Name : .....

M.Sc DEGREE (CSS) EXAMINATION, MAY 2025

Second Semester

M Sc MATHEMATICS

CORE - ME010205 - MEASURE AND INTEGRATION

2019 ADMISSION ONWARDS

EA91D024

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define Lebesgue outer measure. Find the outer measure of the empty set $\emptyset$.
2. Prove that the measurable sets possess excision property.
3. Explain the construction of Cantor set.
4. Prove that a monotone function defined on an interval is Lebesgue measurable
5. Define a simple function and its canonical form. State the simple approximation lemma and simple approximation theorem.
6. Let $\{E_i\}_{i=1}^n$ be a finite disjoint collection of Lebesgue measurable subsets of a set of finite measure E. Prove that if a_i be a real number for $1 \leq i \leq n$ and $\phi = \Sigma_{i=1}^n a_i . \chi_{E_i}$ on E, then $\int_E \phi = \Sigma_{i=1}^n a_i . m(E_i)$.
7. Define a general measure. Show that the counting measure defined on some set satisfies the properties of a measure.
8. When will you say that two measures are mutually singular? Give an example.
9. *Let $(X, \mathcal{M}, \mu)$ be a measure space and ϕ and ψ be nonnegative simple function on X. If ϕ and ψ are positive real numbers and if α and β are positive real numbers, then prove that*
$$\int_X [\alpha\psi + \beta\phi]\, d\mu = \alpha \int_X \psi\, d\mu + \beta \int_X \phi\, d\mu$$
10. State and prove a necessary condition for a bounded measurable function f on a measure space to be integrable?

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. If $\{E_k\}_{k=1}^{n}$ is any finite collection of measurable sets, then prove that $\bigcup_{k=1}^{n} E_k$ is measurable.

12. Define Lebesgue measure. Prove that Lebesgue measure is countably additive.

13. State and prove Bounded Convergence Theorem.

14. State and prove Fatou's Lemma for nonnegative Lebesgue measurable function on E.

15. Prove that the restriction of general outer measure to the collection of measurable sets is finitely additive.

16. Let $(X, \mathcal{M}, \mu)$ be a complete measure space. If f and g are extended real valued functions on X for which $f = g$ a.e. on X, prove that f is measurable if and only if g is measurable.

17. Let $(X, \mathcal{M}, \mu)$ be a measure space and $\{f_n\}$ an increasing sequence of nonnegative measurable functions on X. Define $f(x) = \lim_{n\to\infty} f_n(x)$ for each x in X. Then prove that
$\lim_{n\to\infty} \int_X f_n \, d\mu = \int_X f \, d\mu$

18. State Tonelli's Theorem.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19.
1. Define algebra and σ-algebra.
2. Prove that the collection $\mathcal{M}$ of measurable sets is a σ-algebra that contains the σ-algebra $\mathcal{B}$ of Borel sets.
3. Prove that every G_δ sets and F_σ sets are measurable.

20. State and prove the properties of Linearity, Monotonicity and Additivity over domains of integration for Lebesgue integration of bounded Lebesgue measurable functions on a set of finite measure E

21. Let ν be a signed measure on the general measurable space $(X, \mathcal{M})$. Prove that there exists two unique pair of mutually singular measures ν^+ and ν^- such that $\nu = \nu^+ - \nu^-$.

22. (a) State Radon Nikodym Theorem
(b) Write a short note on Radon Nikodym Derivative.
(c) State Lebesgue Decomposition theorem

(2×5=10 weightage)